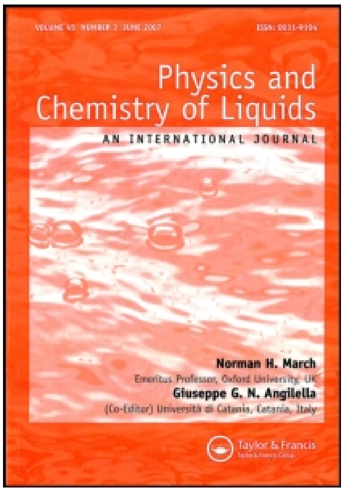

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ON LAMINAR BOUNDARY LAYER FLOW OF ELECTRICALLY CONDUCTING LIQUIDS NEAR AN ACCELERATED VERTICAL PLATE

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The unsteady hydromagnetic flow of electrically conducting liquids whose Prandtl numbers are different from unity has been considered when the flow takes place near an infinite vertical flat plate subject to uniform heat flux and accelerated motion. A unified exact solution has been derived for the boundary layer velocity and skin friction for the cases of magnetic field being fixed relative to the fluid or to the vertical plate. The solution has been presented in real forms for fluids whose Prandtl numbers are greater than or less than unity. The response of the boundary layer fluid velocity to the variations in magnetic and buoyancy forces has been discussed for two sample fluids corresponding to the different Prandtl number categories. The influence of these forces on the skin friction has also been shown.

Keywords: Hydromagnetic flow; Heat flux; Buoyancy force; Free convection; Skin friction

1. INTRODUCTION

In the study of unsteady boundary layer flows of fluids, one of the most important aspects of the study concerns the investigation of the response of the boundary layer to externally applied forces.

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Among the external forces, the forces of gravity and magnetic field are known to play dominant roles in flow control and design of equipments. Hydromagnetic boundary layers are encountered in various industrial and technological applications employing liquid metal and plasma flows subject to transverse magnetic fields [1,2]. Fluid flow controls through external magnetic forces are also used in aerospace industry, chemical engineering, nuclear reactors and geothermal engineering, among others. Moreover, when the fluid flow is generated by moving boundaries, the problems are of great research interest because of their wide industrial applications.

As is known, when the flow field comprises an electrically conducting fluid under the influence of an externally applied magnetic field, the combined effects of the viscous, buoyancy and magnetic forces render the system of Navier-Stokes equations highly non-linear and coupled. For flows near flat plates, these equations can be simplified considerably. However, the degree of difficulty of solution still depends on the physical situation and the assumptions inherent to the problem. Whereas there have been several studies incorporating the non-linear features of the flow phenomena both in the non-magnetic and magnetic cases [3–5], major efforts in literature have also been directed to obtaining analytical solutions of the linear problems. Analytical solutions, though mostly representing idealized situations, are also important partly because of their wider applicability in understanding the basic physics of the problem, and partly because of their possible applications in industrial and technological fields. Of the hydromagnetic fluid flow models amenable to exact analytical treatments, the well-known Stokes and Rayleigh problems in which an infinite flat plate bounding a stationary viscous fluid is moved in its own plane have, for instance, helped in identifying the basic interaction features between the magnetic and fluid dynamical forces. In these problems it can be seen that the quadratic convection terms in the governing equations drop out, and the equations can be reduced to linear systems. However, it must be admitted that the availability of closed form solution for the Stokes problem is still dependent on the nature of the initial and boundary conditions. Further complications arise if one considers the real fluid properties also. For instance, Gebhart *et al.* [6] have given a detailed account of the implications of different types of boundary conditions for convection flows.

The solutions of hydromagnetic free convection flows near plates of infinite extent have been discussed by several authors (see, e.g. [7–12]) under different physical conditions.

As regards the works of direct relevance to the present problem in which the fluid flow is generated by an infinite plate in accelerated motion, Raptis and Singh [10] have obtained solutions for the magnetohydrodynamic free convection flow past an accelerated vertical plate, but their analysis did not consider the heat flux at the boundary. This problem was further extended recently to include the effect of heat flux on the boundary [12], but this work was rather restricted in that the magnetic field was assumed to be fixed relative to the fluid only, and furthermore, the results were presented only for fluids of Prandtl number, P, greater than unity because of the difficulty in obtaining real solutions in exact form to the case when $P < 1$. However, it is of practical interest to understand the relative influence of magnetic field on fluid velocity based on its mode of application. In this respect, the coupling effects of magnetic and buoyancy forces when the magnetic field is applied relative to the moving boundary has not yet been investigated. This problem has thus been taken up in this paper. In fact, in order to make the solution self-contained, and for comparison purposes, an exact solution has been obtained, using Laplace transforms, in a unified form (see Eq. (11) later) encompassing both the cases of magnetic field being fixed to the fluid or to the boundary. It may be remarked that the solution so obtained is more general than the ones existing in literature as it exhibits the combined influence of buoyancy effects, heat flux and the magnetic field including its mode of application. As the heat flux effects entail consideration of derivative boundary conditions, exact solutions of such problems are more difficult to obtain when the velocity field gets coupled with the temperature field. It is also worth mentioning here that the solution obtained in this paper is not applicable for fluids of Prandtl number equal to unity. This case has been considered in detail in [13]. Accordingly, we consider here incompressible fluids of Prandtl numbers different from unity. Our objective in this work thus is threefold: (i) obtain a unified solution to the boundary layer velocity of the free convection flow corresponding to the cases of an external magnetic field being applied fixed relative to the moving fluid or to the moving boundary when the fluid flow is induced by an infinite vertical plate subject to uniform heat flux and accelerated motion; (ii) present the solution so obtained in real forms for the cases of $P<1$ and $P>1$; and (iii) use the real solutions to present some case studies involving liquids of different Prandtl numbers in order to bring out the influence of magnetic and heat flux parameters on their boundary layer motion.

In Section 2, the boundary layer equations for the fluid flow have been presented. These have been solved in Section 3. The solution has been shown to have two different real forms according as the Prandtl number of the fluid is greater than or less than unity. An expression for the skin friction has also been derived. The effects of the magnetic field and the buoyancy force parameters on the boundary layer velocity have been discussed in Section 4 for two sample liquids: mercury and water whose Prandtl numbers (at 20°C) have been taken to be 0.044 and 7.0, respectively.

2. GOVERNING EQUATIONS

As mentioned in the previous section, we consider the unsteady two-dimensional flow of an electrically conducting incompressible fluid of Prandtl number different from unity past an infinite vertical flat plate which is assumed to be non-conducting. With respect to an arbitrarily chosen origin O on this plate, the axis Ox' is taken along the wall in the upward direction and the axis Oy' is taken perpendicular to it into the fluid. For times $t'\leq 0$, the plate and the fluid medium are at rest and at the constant temperature T'_∞. At time $t'>0$, the plate is set into motion with a velocity proportional to t'^n, and simultaneously, heat is also supplied to the plate at a constant rate. The flow takes place under the influence of an external magnetic field of constant strength $(0, B_y, 0)$ applied in the y' direction. Two different flow situations will be considered here with respect to the magnetic field. The first corresponds to the case when the magnetic lines of force are fixed relative to the fluid, and the other when these are fixed relative to the boundary. These two cases will, however, be combined into a single momentum equation so as to obtain a unified solution. As is common in Stokes problems, we assume that the effects of the convective and pressure gradient terms in the momentum and energy equations are negligible. The density of the liquid is assumed to be constant; however, in the case of free convection flow, it is considered variable in forming the buoyancy force. We also assume that the electric and polarization effects in the fluid flow can be neglected. For many fluids used in the laboratory or industrial applications, the electrical conductivity is usually small. This implies that the magnetic Reynolds number is very small so that the induced magnetic field produced by the motion of the electrically conducting fluid is negligible in comparison with the applied one. Moreover, as a result of the boundary layer approximations, the physical variables become functions of the time variable t' and the space variable y' only. Under these assumptions, the boundary layer momentum equation can be written in the form [14]

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) - \frac{\sigma B_y^2}{\rho}(u' - KUt'^n) \tag{1}$$

where u' is the velocity in the x' direction, T' the temperature of the fluid, g the acceleration due to gravity, β the volumetric coefficient of thermal expansion, ν the kinematic viscosity, ρ the density, U a constant and

$$K = \begin{cases} 0, & \text{if } B_y \text{ is fixed relative to the fluid} \\ 1, & \text{if } B_y \text{ is fixed relative to the plate.} \end{cases}$$

The boundary layer energy equation, neglecting viscous dissipation and Ohmic heating, is

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} \tag{2}$$

where k is the thermal conductivity and c_p is the specific heat of the fluid at constant pressure.

The initial and boundary conditions relevant to the fluid flow are

$$\begin{aligned} &u' = 0, \quad T' = 0, && \text{for } y' \geq 0 \text{ and } t' \leq 0 \\ &u' = Ut'^n, \quad \frac{\partial T'}{\partial y'} = -\frac{q}{k} && \text{at } y' = 0 \text{ for } t' > 0 \\ &u' \to 0, \quad T' \to T'_\infty && \text{as } y' \to \infty \text{ for } t' > 0 \end{aligned} \tag{3}$$

where q is the heat flux per unit area at the plate. The flow described by Eqs. (1)–(3) is very general in the sense that it corresponds to a power law velocity of the vertical plate. The features of the motion will therefore depend on the value of n, as it will have significant effects on the search for the exact solution as well as on the dynamics of the flow. In this paper, we shall derive exact solution for the fluid velocity in the case of uniformly accelerated motion of the plate which corresponds to $n=1$.

3. BOUNDARY LAYER VELOCITY

In order to write the governing equations in dimensionless form, it is necessary to introduce appropriate scales of length, time, velocity and temperature. Using a characteristic length scale $L=(\nu^2/U)^{1/3}$, we write

$$\begin{aligned} &y = y'/L, \quad t = \nu t'/L^2, \quad u = Lu'/\nu, \quad T = k(T' - T'_\infty)/(qL) \\ &P = \rho\nu c_P/k, \quad G = qg\beta L^4/\left(k\nu^2\right), \quad m = \sigma L^2 B_y^2/(\rho\nu) \end{aligned} \tag{4}$$

In the above, the dimensionless parameters P, G, and m denote the Prandtl number, Grashof number and the square of Hartmann number, respectively.

Using Eq. (4), Eqs. (1) and (2) can be expressed in the dimensionless forms

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - m(u - Kt) + GT \tag{5}$$

$$\frac{\partial T}{\partial t} = \frac{1}{P}\frac{\partial^2 T}{\partial y^2} \tag{6}$$

The initial and boundary conditions become

$$\begin{aligned} &u = 0, \quad T = 0 \qquad &&\text{for } y \geq 0 \text{ and } t \leq 0 \\ &u = t, \quad \frac{\partial T}{\partial y} = -1 \qquad &&\text{at } y = 0 \text{ for } t > 0 \\ &u \to 0, \quad T \to 0 \qquad &&\text{as } y \to \infty \text{ for } t > 0 \end{aligned} \tag{7}$$

Under the assumptions of the flow problem, we observe that the energy Eq. (6) is uncoupled from the momentum Eq. (5). We can therefore solve for the temperature variable $T(y,t)$ whereupon $u(y,t)$ can be expressed in terms of $T(y,t)$. As stated before, the governing partial differential Eqs. (5) and (6) with the initial and boundary conditions (7) are amenable to an exact analytical treatment using Laplace transforms. Taking Laplace transforms of Eqs. (5) and (6) will result in a set of (ordinary) differential equations for the transformed functions in the independent variable y. On solving, the transformed temperature variable $\bar{T}(y,s)$ can be obtained as

$$\bar{T}(y,s) = \frac{\exp(-y\sqrt{Ps})}{\sqrt{P}s^{3/2}} \tag{8}$$

which, on inversion [15], yields

$$T(y,t) = 2\sqrt{\frac{t}{\pi P}}\exp\left(-\frac{Py^2}{4t}\right) - y\,\mathrm{erfc}\left(\frac{\sqrt{P}y}{2\sqrt{t}}\right) \tag{9}$$

where erfc(x) is the complementary error function defined by

$$\mathrm{erfc}(x) = 1 - \mathrm{erf}(x), \quad \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-\eta^2)\,\mathrm{d}\eta$$

To obtain the solution for the velocity variable, we solve the transformed momentum equation and write the velocity in the (y,s)-plane in the form

$$\bar{u}(y,s) = \bar{u}_1(y,s) + \bar{u}_2(y,s) + \bar{u}_3(y,s) \tag{10}$$

where

$$\bar{u}_1(y,s) = \frac{Km}{(s+m)s^2} + \frac{s+(1-K)m}{(s+m)s^2}\exp(-y\sqrt{s+m})$$

$$\bar{u}_2(y,s) = -\frac{a\exp(-y\sqrt{s+m})}{(s+b)s^{3/2}}, \quad \bar{u}_3(y,s) = \frac{a\exp(-y\sqrt{Ps})}{(s+b)s^{3/2}}$$

$$a = \frac{G}{(1-P)\sqrt{P}}, \quad b = \frac{m}{1-P}$$

The solution for the velocity variable in the physical (y, t)-plane can be obtained by direct inversion (for $\bar{u}_1$) and using convolution (for $\bar{u}_2$ and $\bar{u}_3$) [15,16]. After detailed simplifications, it can be shown that $u(y, t)$ can be expressed in the form

$$u(y, t) = u_1(y, t) + u_2(y, t) + u_3(y, t) + u_4(y, t) \tag{11}$$

where

$$u_1(y, t) = \frac{K}{m}\left[mt - 1 + \varphi_1(y, t) + \varphi_2(y, t) + \exp(-mt)\,\mathrm{erf}\left(\frac{y}{2\sqrt{t}}\right)\right]$$
$$u_2(y, t) = (1 - K)\left[\left(t - \frac{y}{2\sqrt{m}}\right)\varphi_1(y, t) + \left(t + \frac{y}{2\sqrt{m}}\right)\varphi_2(y, t)\right]$$
$$u_3(y, t) = -\frac{2a}{\sqrt{\pi}}\int_0^t [\varphi_3(y, x) + \varphi_4(y, x)]\sqrt{t - x}\,\mathrm{d}x$$
$$u_4(y, t) = \frac{2a}{\sqrt{\pi}}\int_0^t [\varphi_5(y, x) + \varphi_6(y, x)]\sqrt{t - x}\,\mathrm{d}x$$
$$\varphi_{1,2}(y, t) = \frac{1}{2}\exp(\mp y\sqrt{m})\,\mathrm{erfc}\left(\frac{y}{2\sqrt{t}} \mp \sqrt{mt}\right)$$
$$\varphi_{3,4}(y, t) = \frac{1}{2}\exp(-bt \mp iy\sqrt{bP})\,\mathrm{erfc}\left(\frac{y}{2\sqrt{t}} \mp i\sqrt{bPt}\right)$$
$$\varphi_{5,6}(y, t) = \frac{1}{2}\exp(-bt \mp iy\sqrt{bP})\,\mathrm{erfc}\left(\frac{y\sqrt{P}}{2\sqrt{t}} \mp i\sqrt{bt}\right)$$

In the above definitions of $\varphi_{i,\,j}(y, t)$, $(i = 1, 3, 5;\ j = 2, 4, 6)$, the upper sign goes with i and the lower sign with j.

We note that the solution for $u(y, t)$ given by Eq. (11) is not valid when the Prandtl number is equal to unity since both a and b are undefined in this case. The solution distinguishes clearly the cases of magnetic field fixed relative to the fluid ($K = 0$) or fixed relative to the boundary ($K = 1$). It may be noted that in the absence of buoyancy, ($a = 0$), Eq. (11) will correspond to the results in [10] and will also follow as a special case of a related rotating magnetohydrodynamic flow study in [17]. In the absence of magnetic field, ($b = 0$), the solution (11) does not reduce directly to the required solution because of division by zero. Putting $m = 0$ in the momentum Eq. (5), and solving using Laplace transforms, it can be shown that the velocity variable for the non-magnetic case can be expressed as

$$\begin{aligned} u(y,t) = {} & \left(\frac{ay^3}{6} + \frac{y^2}{2} + ayt + t\right)\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right) \\ & - \sqrt{P}\left(\frac{Pay^3}{6} + ayt\right)\operatorname{erfc}\left(\frac{\sqrt{P}y}{2\sqrt{t}}\right) \\ & - \frac{\sqrt{t}}{3\sqrt{\pi}}(ay^2 + 3y + 4at)\exp\left(-\frac{y^2}{4t}\right) \\ & - \frac{a\sqrt{t}}{3\sqrt{\pi}}(Py^2 + 4t)\exp\left(-\frac{Py^2}{4t}\right) \end{aligned} \tag{12}$$

The above solution agrees with a special case of an earlier work on mass transfer and heat flux effects [18] at an accelerated vertical plate. In the special case of the magnetic field fixed relative to the fluid, Eq. (11) will reduce to the results in [12], although the case $P<1$ was not discussed therein because of the computational difficulties. However, the study of the class of fluid flows belonging to this category of Prandtl numbers is of importance in several applications. We shall therefore present here explicit real expressions for the boundary layer velocity for all values of Prandtl number except unity. We observe that the analytical solution (11) will appear in different forms depending on whether the Prandtl number is greater than or less than unity since $b>0$ when $P<1$ and $b<0$ when $P>1$. In particular, the exponential and the complementary error functions in Eq. (11) will have real arguments when $P>1$ and complex arguments when $P<1$. Thus the solution as appearing in Eq. (11) is the real form of the velocity for fluids of Prandtl number greater than unity. When $P<1$, it is necessary to express $u(y,t)$ in real form for computational purposes. This in turn necessitates separating $\operatorname{erfc}(x+iy)$ into real and imaginary parts. To this end, for (x,y) in the first quadrant, we write [19]

$$\operatorname{erfc}(x+iy) = f(x,y) + ih(x,y) \tag{13}$$

where

$$f(x,y) = \sum_{n=0}^{\infty}\left[(xy)^{2n}g_n(x)\cos 2xy - (n+1)(xy)^{2n+1}g_{n+1}(x)\sin 2xy\right]$$

$$h(x,y) = -\sum_{n=0}^{\infty}\left[(xy)^{2n}g_n(x)\sin 2xy + (n+1)(xy)^{2n+1}g_{n+1}(x)\cos 2xy\right]$$
$$g_{n+1}(x) = \frac{2}{2n+1}\left[\frac{\exp(-x^2)}{\sqrt{\pi}(n+1)!\,x^{2n+1}} - \frac{g_n(x)}{n+1}\right]$$
$$g_0(x) = \mathrm{erfc}(x)$$

Alternatively, we may also use the approximation for $\mathrm{erfc}(x+iy)$ as given in [15] with the corresponding definitions of the functions $f(x,y)$ and $h(x,y)$. However, this will yield only approximate real solutions.

In Eq. (11), we note that only $u_3(y,t)$ and $u_4(y,t)$ have complex arguments when $P<1$, but $u_1(y,t)$ and $u_2(y,t)$ do not change with P. Accordingly, using Eq. (13), we may express the real forms of $u_3(y,t)$ and $u_4(y,t)$ as

$$u_{3,4}(y,t) = \mp\frac{2a}{\sqrt{\pi}}\int_0^t [f_{3,4}(x,y) - h_{3,4}(x,y)]\exp(-bx)\sqrt{t-x}\,\mathrm{d}x \quad (14)$$

where the first subscript goes with the upper sign, the second with the lower sign, and

$$f_3(x,y) = f\left(\frac{y}{2\sqrt{x}}, \sqrt{bPx}\right)\cos(y\sqrt{bP})$$
$$h_3(x,y) = h\left(\frac{y}{2\sqrt{x}}, \sqrt{bPx}\right)\sin(y\sqrt{bP})$$
$$f_4(x,y) = f\left(\frac{y\sqrt{P}}{2\sqrt{x}}, \sqrt{bx}\right)\cos(y\sqrt{bP})$$
$$h_4(x,y) = h\left(\frac{y\sqrt{P}}{2\sqrt{x}}, \sqrt{bx}\right)\sin(y\sqrt{bP})$$

Eq. (14) will be used in Eq. (11) for evaluating the convolution integrals when $P<1$.

Skin Friction

In order to evaluate the shear stress at the boundary, we consider the skin friction $\tau(= -\partial u/\partial y|_{y=0})$. On differentiating Eq. (11) with respect to y, the skin friction can be derived in the form

$$\tau(t) = \tau_1(t) + \tau_2(t) + \tau_3(t) \tag{15}$$

where

$$\begin{aligned}
\tau_1(t) &= \frac{K}{\sqrt{m}}(1 - \operatorname{erfc}\sqrt{mt}) \\
\tau_2(t) &= (1 - K)\left[\left(\frac{1 + 2mt}{2\sqrt{m}}\right)(1 - \operatorname{erfc}\sqrt{mt}) + \sqrt{\frac{t}{\pi}}\exp(-mt)\right] \\
\tau_3(t) &= \frac{2a}{\sqrt{\pi}}\int_0^t \sqrt{t - x}\,\gamma(x)\,\mathrm{d}x \\
\gamma(x) &= \frac{1}{\sqrt{\pi x}}\left[\sqrt{P} - \exp(-mx)\right] \\
&\quad + i\sqrt{bP}\exp(-bx)\left[\operatorname{erfc}(i\sqrt{bPx}) - \operatorname{erfc}(i\sqrt{bx})\right]
\end{aligned}$$

In the non-magnetic case, the skin friction can be obtained using Eq. (12). In this case we obtain

$$\tau(t) = a(\sqrt{P} - 1)t + 2\sqrt{t/\pi} \tag{16}$$

4. NUMERICAL RESULTS

Our aim in this work has been to obtain an analytical solution for the hydromagnetic boundary layer flow of an incompressible fluid when the flow takes place due to the accelerated motion of a vertical plate subject to uniform heat flux. The solution is applicable to the cases of the external magnetic field applied fixed relative to the fluid or to the plate. The exact solution thus obtained has been used in this section to investigate numerically the influence of the external forces. We have computed the fluid velocity given by Eq. (11) for two sample liquids: mercury and water, whose Prandtl numbers at 20°C have been taken to be 0.044 and 7.0, respectively. As mentioned in the previous section, the velocity of mercury for which P is less than unity has been evaluated using Eqs. (11) and (14).

In Fig. 1 we have shown the variation of velocity of mercury in the boundary layer for some values of the magnetic and buoyancy parameters, while Fig. 2 shows the velocity profiles of water for another set of values. The curves correspond to the case of magnetic field fixed relative to the boundary. From Figs. 1 and 2 we observe two different kinds of fluid flow behaviour depending on the magnitude of the Grashof number G. For relatively small values of G, the fluid velocity decreases steadily from the plate velocity to its zero free-stream value. However, for higher values of G, the fluid velocity overshoots the plate velocity in regions close to the boundary. This overshooting is more pronounced for low Prandtl number fluids than for higher Prandtl number fluids. This is evident from Fig. 1 for mercury in which a small increase in G has been shown to result in a large increase in its velocity near the boundary, whereas in the case of water (see Fig. 2), even a very large increase in G resulted in only a relatively small overshooting of u near the boundary.

With regard to the relative influence of the magnetic field being fixed relative to the fluid or to the plate, although not shown in the figures, it was found that they had opposite effects on the fluid flow in the boundary layer. In general, the fluid velocity was seen to decrease with increase in magnetic field strength when it was applied

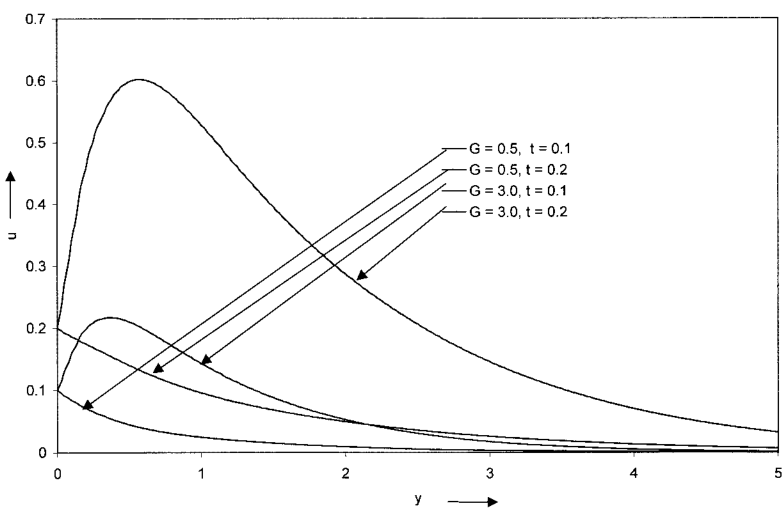

FIGURE 1 Velocity u of mercury ($P = 0.044$, $m = 0.1$, $K = 1$).

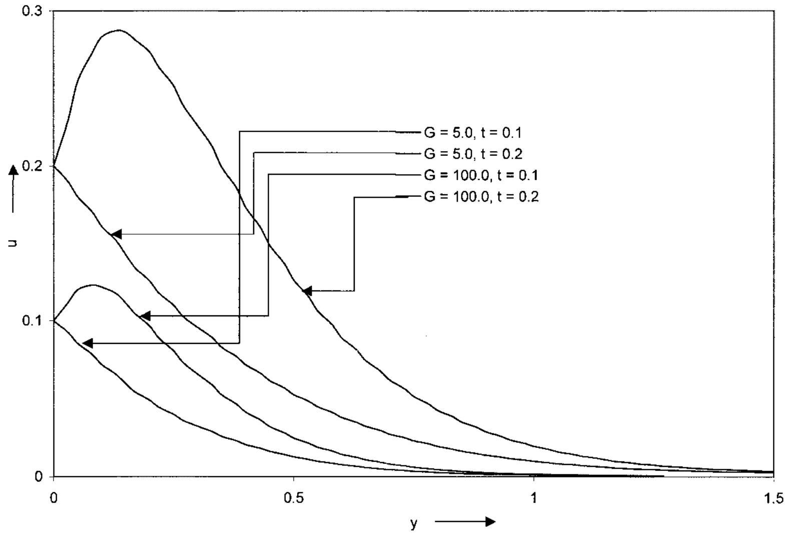

FIGURE 2 Velocity u of water ($P=7.0$, $m=0.1$, $K=1$).

fixed relative to the fluid, and the opposite effect was observed when the magnetic field was fixed relative to the plate. The qualitative change in the velocity profiles is a consequence of the force exerted by the magnetic field on the fluid. This feature was seen to be valid for different values of G and t. However, within the scales chosen and for the specific parameter values considered, the values of u differed only by small amounts for the $K=0$ and $K=1$ cases; therefore, the curves could not be shown distinctly in the figures.

The influence of the magnetic and buoyancy parameters on the skin friction τ under different temporal domains has been shown in Table I. For illustrative purposes, we have shown the skin friction for water only. As in the case of velocity, the magnetic field has opposite effects on the skin friction depending on the mode of application. τ increases or decreases with m according as the magnetic field is applied fixed relative to the fluid or to the boundary, respectively. For a fixed m, the skin friction has a larger value when $K=0$ than when $K=1$, indicating an increased rate of shear at the boundary wall in the former case due to the interaction between the magnetic and viscous forces. The skin friction decreases with the Grashof number G for all other

TABLE I Skin friction τ of water ($P = 7.0$)

G	t	m	τ	
			$K=0$	$K=1$
0.5	0.1	0.1	0.3535	0.3511
		1.0	0.3641	0.3408
	0.3	0.1	0.6108	0.5985
		1.0	0.6649	0.5483
5.0	0.1	0.1	0.3131	0.3107
		1.0	0.3240	0.3007
	0.3	0.1	0.4897	0.4774
		1.0	0.5464	0.4297
20.0	0.1	0.1	0.1784	0.1761
		1.0	0.1903	0.1670
	0.3	0.1	0.0863	0.0740
		1.0	0.1513	0.0346

parameter values. The temporal variation of τ was seen to be not monotonic; to a large extent it is influenced by the magnitude of G.

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